OPINION

Curating Mathematics for the 21st Century
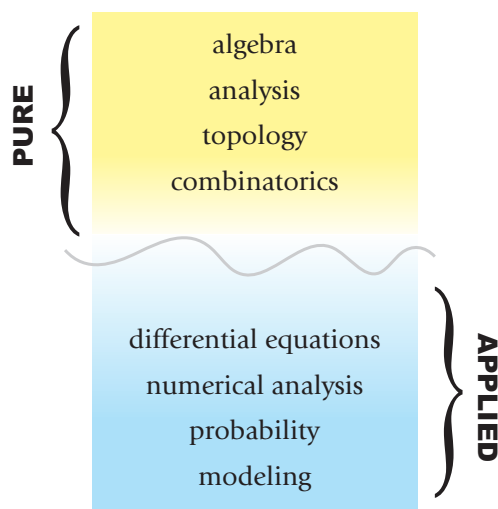
Satyan L. Devadoss

1. Classification and Curation

Today, our discipline of mathematics is roughly partitioned into two groups, pure and applied, though there has never been consensus on the meaning of 'applied' mathematics. During the 17th and 18th centuries, there was an equally vague and equally complex division in mathematics between *mathematicae purae* (pure) and *mathematicae mixtae* (mixed). The former dealt with ideas distinct from matter (such as geometry and arithmetic) whereas the latter overlapped with areas such as music, astronomy, and architecture [3]. In the 19th century, strongly influenced by the French *Encyclopédie* of Diderot and d'Alembert, the notion of 'applied' replaced 'mixed,' with applications (especially in fields such as hydrodynamics and mechanics) exerting a strong influence on the nomenclature [7].

With the advent of modeling methods, the 20th century saw applications rapidly gaining importance across numerous disciplines, and today, applied mathematics has developed into a sophisticated area yielding powerful tools and results, impacting nearly all aspects of industry, economics, medicine, technology, and the sciences. This maturity has resulted in the codification of subjects that appear in its pedagogical canon, including ODE, PDE, numerical analysis, probability, and modeling, with pure mathematics

Satyan L. Devadoss is the Fletcher Jones Professor of Applied Mathematics and Professor of Computer Science at the University of San Diego. His email address is devadoss@sandiego.edu.

covering areas such as combinatorics, algebra, analysis, topology, and geometry. These areas are certainly not without overlap, and the partition not always standard.

Yet the distinctions between these (already porous) categories are swiftly fading. After all, where is the line between linear algebra and differential equations and modern geometry? Or between probability and combinatorics and algebra? And the remarkable results over the past decades[1] have brought to light profound connections and interplays between pure and applied areas, nearly eliminating any demarcation. Added to this is a curious matter taking place in the 21st century: the *applications* of pure mathematics to real-world situations have become prolific. One clear example is knot theory: a relatively specialized and

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[1]*Just looking at recent works of Fields Medalists provides significant clues.*

deeply theoretical subfield of topology 50 years ago, it now impacts quantum mechanics, polymer chemistry, string theory, DNA entanglement, and cosmology [8]. This phenomenon is not unique to topology but occurs across the entire spectrum of pure mathematics: algebraic geometry to phylogenetics, homotopy theory to data analysis, complex analysis to signal processing, and (of course) number theory to cryptography. My own work as a trained topologist has found applications from the phylogeny of beer and flexible architecture to computational cartography and polyhedral sculptures.

If it is true that the pure/applied divide is no longer the right way to slice the mathematical pie, it might be tempting to remove distinctions altogether and categorize everything as just 'mathematics.' We might want to simply agree with V. I. Arnol'd who famously wrote [2]:

> Mathematics is the part of physics where experiments are cheap.

Yet classifications serve an important purpose. A museum curator designs an exhibit by grouping artworks in order to impart a perspective and a story for its viewers. Our work as mathematicians warrants a similar calling: to frame our work to offer greater clarity and better access to mathematics, an invitation to students and the larger community. Indeed, this pure/applied division, having been codified and centralized through academic journals and departmental structures, impacts all of us at nearly every stage of our academic life: hiring decisions, salary levels, grant opportunities, curricular offerings, undergraduate degrees, departmental reviews, and faculty mentoring. The changes over the past few decades beckon us to reevaluate our discipline, for the current delineation no longer serves its purpose.

2. Lessons from Biology

In order to better understand the transformation occurring in mathematics today, we consider a crisis encountered in biology around 50 years ago. In the mid-20th century, biology was broken into natural divisions based on *taxonomy*, the classification of organisms and their relationship to the environment. A classical example of this segmentation was between botany (study of plants) and zoology (study of animals). Near the end of the 20th century, however, the framework using taxonomy was mostly abandoned for one that highlighted *process*. Now divisions are based on how something functions (molecular biology, cell biology, ecology, evolution) rather than where it belongs in the tree of life.

Mathematics is headed in a similar trajectory: With the tremendous growth in data acquisition and analysis, the 21st century has brought about a phenomenal resurgence in mathematics through a *computational* lens. The pure versus applied partition has given way to theory versus computation. With the advent of new algorithms
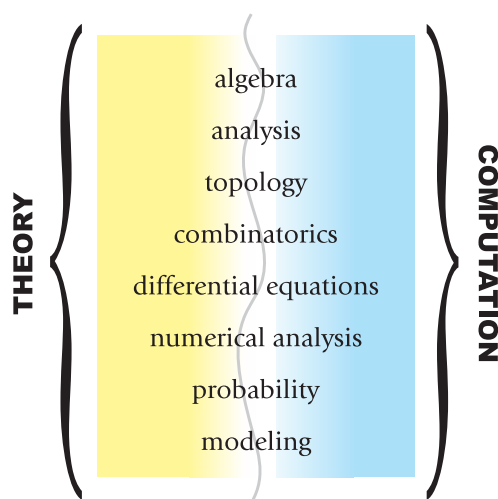
powered by technological advances making possible the study of increasingly large and complex systems, computational thinking has become a challenger to the theoretical path towards mathematics creation. The sciences, from physics to biology to chemistry, have already experienced and embraced this computational shift. It is now our turn.

An algorithmic approach to mathematics is not new, and calculations of all kinds have been a marker across our discipline. What is distinct now is that computational tools (including machine learning) are yielding rich results on par with theoretical methods, and these results are occurring across *all* mathematics. This impact on the 'applied' mathematics canon has already been so profound that some have argued it to have opened "a new era in applied mathematics" [6]. But disciplines under the 'pure' umbrella have also been forcibly transformed, from geometry (computational origami, mesh generation), topology (persistent homology, topological data analysis), and algebra (discrete optimization, SageMath software). Moreover, machine-learning methods are appearing in pure mathematics through discoveries of new conjectures and theorems in knot theory to group theory [5]. Even proof construction (with the 4-color theorem as forebearer) and proof-checking have become automated [9] and accepted by the community, and acutely abstract fields such as category and homotopy type theory are aiding in computations that provide confidence in theory correctness [14].

A great benefit to this process-based approach to mathematics curation is that it is *discipline independent*: every subfield of mathematics can play the game, whether in theory or computation. It also communicates a clear message to students seeking a home, who thirst for a computational bent to see the world. A curricular approach to this new curation could be crudely sketched as follows:

Opening Overlap: An introduction to mathematics for both theoretical and computational tracks should include exposure to *continuous* (calculus, differential equations) and *discrete* (sets, numbers, statistics) points of view. The

ubiquitous nature of data has made it clear that discrete tools are no longer relegated to the margins but should take center stage again. Similarly, the language of proofs should be presented and developed on equal grounds with the analysis of algorithmic and data-driven methods.

Theoretical Track: The full spectrum of mathematics courses is available here for exploration, including PDE, numerical analysis, and probability. However, these topics would be framed primarily from a theoretical lens (existence, asymptotics and bounds, extending results to infinite dimensions). Advanced courses in algebra, for instance, might be presented classically (Atiyah-MacDonald comes to mind), and geometry would be differential and Riemannian.
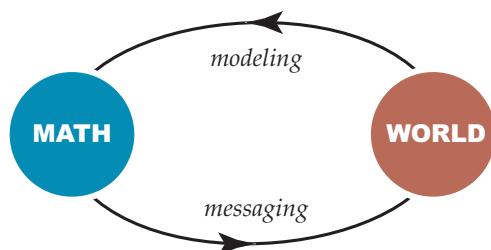
Computational Track: The full spectrum of mathematics courses is available here as well, but approached from a computational lens. This could be straightforward in some fields (numerical methods, stochastic analysis) but novel in others. For example, geometric concepts could be viewed discretely (triangulations, simplicial complexes, Cauchy rigidity) since discretization aids in preparation for computations. And advanced algebra might be presented as in a recent book by Michalek–Sturmfels [11], covering representation theory, tropical algebra, and Schubert calculus.

Closing Overlap: Since the applications of mathematics to physical and practical situations have become bountiful, both tracks should require *modeling* (through graph theory or PDE, for instance), bringing ideas of the world into the mathematical realm. Equally important would be concepts in *messaging*: techniques of bringing mathematics to the world at large. Opportunities to present and showcase all types of mathematics (spoken word, written essay, illustrative graphic) to corporations, government agencies, think tanks, and a thirsty public are proliferating, highlighting further the importance of proper presentation.

Though this sketch is a clumsy starting point, it foreshadows a larger discussion in our mathematics community. Biology evolved into a process-based framing, impacting every aspect of its field, exploding in impact and importance in the 21st century. The developments over the past few decades show the need to reevaluate our own discipline. The sooner we embrace this change, the better.

3. Accountability and Humanity

A word of warning: there is a tradition that claims that unlike the sciences, mathematics does not need to entangle itself in the petty affairs of the world. Bertrand Russell expressed this when he penned these words over a century ago [13]:

> *Remote from human passions, remote even from the pitiful facts of nature, the generations have gradually created an ordered cosmos where pure thought can dwell as in its natural home, and where one, at least, of our nobler impulses can escape from the dreary exile of the actual world.*

Of course, regardless of their level of abstraction, almost all mathematical theories eventually become applied to the real world. But even in the most generous of situations, the impact of theoretical mathematics has been measured in decades if not centuries.

With the embrace of computational techniques, this timescale has become compressed tenfold. Even the most abstract mathematician can no longer be divorced from the world. Our need for data and its acquisition will only increase in the 21st century and we need to be prepared for the consequences that come with technological entanglements. The positive influences of mathematics on society at large have been thoroughly ingrained into the fabric of our community.[2] Yet there are darker ramifications of our work, and mathematicians cannot reap the rewards without facing the consequences. We name a few:

(1) Since growth and drive towards computational thinking is data driven, the collection methods of this data are of real concern. Large technology companies are leveraging machine-learning tools to acquire information on nearly every aspect of our lives, from location tracking to facial expressions, a form of "surveillance capitalism" [18]. There are also worries emanating from the ways this data is being used, from targeted market saturation to law enforcement tactics in urban settings, notably against the disenfranchised. All of these form threats against equity and democracy that big data invites [12].

(2) Moreover, the very nature of some computational techniques leads to *algorithm bias*: the inequities of the past are both incorporated into and repeated by model fits, which in turn base predictions off such data. This appears even in our own realm of higher education, in the form of learning analytics and grade forecasting [10]. These algorithms seem to undermine rather than promote the student success we desire.

(3) Another area of concern is the magnitude of hidden environmental costs associated with computational

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[2]*Consider the* Broader Impacts *portion of a National Science Foundation grant requirement, for example.*

tasks. Though we speak of abstract algorithms and disembodied cloud computing, none of it can function without concrete materials and resources. Exorbitant quantities of minerals need to be excavated from the earth, along with the associated geopolitical conflicts for mining rights. Moreover, massive amounts of energy are required for servers and data centers, leading to ecological degradation [4]. We are far from the days of chalkboards and pencils and paper.

(4) There are also dangers in how technology affects us as humans. Scholars such as Sherry Turkle [17] warn of loneliness and distraction and philosophers such as Charles Taylor [16] caution against a loss of purpose in this digital age. Equally worrisome are the treatment of humans behind the automation scenes, often performing rote tasks under workplace surveillance treated like the robots they are trying to replace.

As we expect our students to have a robust understanding of computational methods, they need to have a robust understanding of its implications as well. Offering a course or two on ethics is not the solution; instead, an integrated approach to mathematical responsibility is warranted. Today, our community, more than ever, is accountable to the morality of mathematical endeavors and the stewardship of our world. This is part of a larger call for doing mathematics as a means of human flourishing, led by mathematicians such as Federico Ardila-Mantilla [1] and Francis Su [15]. And a curation of our mathematical world, brought about by this new computational mindset, has made the situation all the more pressing.

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Satyan L. Devadoss

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